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# GENERALIZED HECTIC RADICAL FUNCTIONAL EQUATION

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Abstract. In the field of functional equations, till the present level of its developments many researchers obtainded a functional equations satisfied by only upto $f(x) = x^{40}$. In this paper, authors introduced a hectic radical functional equation satisfied by $f(x) = x^{100}$ and investigate its generalized Hyers-Ulam-Rassias($HUR$) stability in probabilistic modular(PM) space by using fixed point theory.

## 1. Introduction and Preliminaries

The stability problem of functional equations originates from the fundamental question: When is it true that a mathematical object satisfying a certain property approximately must be close to an object satisfying the property exactly?

In connection with the above question, in 1940, S. M. Ulam [26] raised a question concerning the stability of homomorphisms. Let $G$ be a group and let $G'$ be a metric group with $d(.,.)$. Given $\epsilon > 0$ does there exist a $\delta > 0$ such that if a function $f : G \to G'$ satisfies the inequality $d(f(xy), f(x)f(y)) < \delta$ for all $x, y \in G$, then there is a homomorphism $H : G \to G'$ with $d(f(x), H(x))$ for all $x \in G$?

The first partial solution to Ulam's question was given by D. H. Hyers [6]. In 1978, Th. M. Rassias [22] provided a generalized version of the theorem of Hyers which permitted the Cauchy difference to become unbounded. The phenomenon that was introduced and proved by Th. M. Rassias is called the Hyers-Ulam-Rassias stability.

In 1982-1989, J. M. Rassias [18, 19] introduced Ulam-Gavruta-Rassias stability involving a product of different powers of norms. Also, very recently J. M. Rassias [21] introduced Hyers Ulam J.M.Rassias stability involving mixed product of powers of norms. In 1994, a generalization of all the above stabilities was obtained by P. Găvruta [5] is called the generalized Hyers-Ulam-Rassias stability.

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Now the authors have obtained an special kind of equation which shall possess the nature of hectic type of functional equation. The equation can be expressed as

$$\begin{aligned} f(\alpha x+y)+f(\alpha x-y)=\alpha^3\left[f(x+y)+f(x-y)\right] \\ +2\left(\alpha^{100}-\alpha^3\right)f(x)+2\left(1-\alpha^3\right)f(y) \\ +\sum_{k\in 2\mathbb{N}}^{98}2(100C_k)\left(\alpha^{100-k}-\alpha^3\right)f\left(\sqrt[100]{x^{100-k}y^k}\right) \end{aligned} \tag{1}$$

for $\alpha\neq 0,\pm 1$ and $k\in\mathbb{N}$.

The notion of metric space was generalized by the author Menger[8] in 1942 by the name of statistical metric space later is now called probabilistic metric space. One can also refer[24]. Later the probabilistic metric space was used by many researchers, see[2, 7, 17, 9, 25, 4]. In 2007, the author K. Nourouzi[16] introduced probabilistic modular to investigate some basic facts in the probabilistic metric space and is now called probabilistic modular space. The orthogonal modular stability of radical quintic functional equation was investigated by the authors R.Murali and P. Divyakumari [10] in 2019. In 2012, stability of a cubic functional equation was investigated by the authors K. Ravi, J.M.Rassias and P. Narasimman [23] in Menger probabilistic normed space.

In 2013, authors Yeol Je Cho et al.[29], presented a fixed point method to prove the generalized Hyers-Ulam stability of additive-quadratic-cubic functional equations in $\beta-$homogeneous probabilistic modular spaces

In 2013, the authors S.Zolfaghari et al.[30] investigated the generalized Hyers-Ulam-Rassias stability of an mixed type functional equation of the form

$$h(\alpha+a\beta)+h(\alpha-a\beta)=h(\alpha+\beta)+h(\alpha-\beta)+\frac{2(\alpha+1)}{\alpha}h(\alpha y)-2(\alpha+1)h(y)$$

for $\alpha\neq 0,\pm 1$.

In 2019, Rassias, Dutta and Narasimman [15] investigated the stability of general $A$-quartic functional equations in non-Archimedean intuitionistic fuzzy normed spaces. Very recently, Murali, Divyakumari and Dutta [13] introduced an Euler-Lagrange radical functional equation with solution and stability. Also, one can refer [1, 11, 12, 14].

**Definition 1.** Let $X$ be a real vector space and if a mapping $\rho : X\to\Delta$ fulfills the following conditions

(i) $\rho(x)(0)=0$,

(ii) $\rho(x)(t)=1$ for all $t>0$, if and only if $x=\Gamma$ ($\Gamma$ is the null vector in $X$),

(iii) $\rho(-x)(t)=\rho(x)(t)$,

(iv) $\rho(\alpha x+\beta y)(r+t)\geq\rho(x)(r)\wedge\rho(x)(t)$,

for all $x,y\in X$, $\alpha,\beta,r,t\in\mathbb{R}^+$, $\alpha+\beta=1$, then a pair $(X,\rho)$ is called a probabilistic modular space and $(X,\rho)$ is $\beta-$homogeneous if $\rho(\alpha x)(t)=\rho(x)(\frac{t}{|\alpha|^\beta})$ for all $x\in X, t>0$, $\alpha\in\mathbb{R}\backslash\{0\}$. Here, $\Delta$ is $f:\mathbb{R}\to\mathbb{R}^+$ the set of all non-decreasing functions with $\inf_{t\in\mathbb{R}}f(t)=0$ and $\sup_{t\in\mathbb{R}}f(t)=1$. Also, the function min is denoted by $\wedge$.

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**Example 2.** Let $X$ is a real vector space and $\rho$ is a modular on $X$. Then a pair $(X,\rho)$ is a probabilistic modular space, where

$$\rho(x)(t) = \begin{cases} \frac{t}{t+\rho(x)}, & t>0 \quad , x\in X \\ 0, & t\leq 0 \quad , x \in X. \end{cases}$$

**Definition 3.** Let $(X,\rho)$ be a probabilistic modular space. Then

(i) A sequence $\{x_n\}$ in $(X,\rho)$ is said to be $\rho-$convergent to $x$, if for all $t>0$ and $\lambda\in(0,1)$, there exists $n_0$ a positive integer such that $\rho(x_n-x)(t) > 1-\lambda$ for all $n \geq n_0$.

(ii) A sequence $\{x_n\}$ in $(X,\rho)$ is said to be $\rho-$Cauchy, if for all $t>0$ and $\lambda\in(0,1)$, there exists $n_0$ a positive integer such that $\rho(x_n-x_m)(t) > 1-\lambda$ for all $n,m \geq n_0$.

(iii) In $(X,\rho)$, every $\rho-$convergent sequence is a $\rho-$Cauchy sequence. If every $\rho-$Cauchy sequence is $\rho-$convergent sequence, then $(X,\rho)$ is called a $\rho-$complete probabilistic modular space.

(iv) $(X,\rho)$ possesses Fatou property if for any sequence $\{x_n\}$ of $X$, $\rho-$converging to $x$, we have $\rho(x)(t) \geq \lim_{n\geq 1}\sup \rho(x_n)(t)$ for all $t>0$.

**Definition 4.** A probabilistic modular $\rho$ is said to satisfy the $\Delta_2-$condition if there exists $\kappa>0$ such that $\rho(2z)=\kappa\rho(z)$ for all $z\in Z$.

The chapter structured as follows: In Section-1 the authors provides necessary introduction of this chapter. In Section-2 the authors obtain general solution of the hectic radical functional equation (1). In Sections-3, the authors discuss generalized Hyers-Ulam-Rassias stability of hectic radical functional equation (1) in probabilistic modular(PM) space using fixed point theory and the conclusion given in Section-4.

## 2. General solution of hectic radical functional equation

In this section author obtains the general solution of a hectic functional equation(1) of the form

$$\begin{aligned} &f(\alpha x+y)+f(\alpha x-y)-\alpha^3 f(x+y)-\alpha^3 f(x-y) \\ &\quad = \left(2\alpha^{100}-2\alpha^3\right) f(x) + \left(2-2\alpha^3\right) f(y) \\ &\quad + \left(2(100C_2)\alpha^{98}-2\alpha^3(100C_2)\right) f\left(\sqrt[100]{x^{98}y^2}\right) \\ &\quad + \left(2(100C_4)\alpha^{96}-2\alpha^3(100C_4)\right) f\left(\sqrt[100]{x^{96}y^4}\right) + \dots \\ &\quad + \left(2(100C_{98})\alpha^{2}-2\alpha^3 100C_{98}\right) f\left(\sqrt[100]{x^{2}y^{98}}\right), \end{aligned} \tag{2}$$

for a fixed real $\alpha$ and $\alpha\neq 0,\pm 1$.

**Theorem 5.** *Let $X$ and $Y$ be real vector spaces. If a mapping $f:X\to Y$ satisfies the functional equation (2), then $f$ is hectic and even.*

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*Proof.* Consider $f$ satisfies the hectic functional equation(2). Setting $(x, y)$ by $(0, 0)$ and $(x, 0)$ in (2), we obtain $f(0) = 0$ and $f(\alpha x) = \alpha^{100} f(x)$, respectively. $\forall x \in X$. Hence, $f$ is hectic.

Substituting $x = 0$ in (2), we arrive $f(-y) = f(y)$, for all $y \in X$ and hence $f$ is even. □

### 3. Stability of hectic radical functional equation in $PM-$space

In this section, author obtains the generalized Hyers-Ulam-Rassias stability of hectic functional equation (1) in probabilistic modular space(PM-space) using fixed point technique. For mapping $f : E \to (X, \rho)$, consider

$$\begin{aligned} M_e(x, y) &= f(\alpha x + y) + f(\alpha x - y) - \alpha^3 f(x + y) - \alpha^3 f(x - y) \\ &- \left(2\alpha^{100} - 2\alpha^3\right) f(x) - \left(2 - 2\alpha^3\right) f(y) \\ &- \left(2(100C_2)\alpha^{98} - 2\alpha^3(100C_2)\right) f\left(\sqrt[100]{x^{98}y^2}\right) \\ &- \left(2(100C_4)\alpha^{96} - 2\alpha^3(100C_4)\right) f\left(\sqrt[100]{x^{96}y^4}\right) - \dots \\ &- \left(2(100C_{98})\alpha^2 - 2\alpha^3 100C_{98}\right) f\left(\sqrt[100]{x^2y^{98}}\right), \end{aligned}$$

for a fixed real $\alpha$ and $\alpha \neq 0, \pm 1$.

**Theorem 6.** *Let $E$ be a linear space, $X$ be a real vector space and $(X, \rho)$ is a $\rho-$complete $\beta-$homogeneous $PM-$space. If a mapping $f : E \to (X, \rho)$ satisfies an inequality of the form*

$$\rho(M_e(x, y)) \geq \nu(x, y)(t), \tag{3}$$

*for all $x, y \in E$ and a given function $\nu : E \times E \to \Delta$, where $\Delta$ is the set of all non-decreasing function such that*

$$\nu(\alpha^a x, 0)(\alpha^{100\beta a} N t) \geq \nu(x, 0)(t) \tag{4}$$

*for all $x \in E$ and*

$$\nu(\alpha^{am} x, \alpha^{am} y)(\alpha^{100\beta am} t) = 1 \tag{5}$$

*for all $x, y \in E$ and a constant $0 < N < \frac{1}{2^\beta}$. Then there exists a unique hectic mapping $M : E \to (X, \rho)$ satisfies (2) and*

$$\rho(M(x) - f(x))\left(\frac{t}{\alpha^{100\beta} N^{\frac{a-1}{2}}(1 - 2^\beta N)}\right) \geq \nu(x, 0)(t) \tag{6}$$

*for all $x \in E$.*

*Proof.* Letting $y = 0$ in (3), we obtain

$$\rho(2f(\alpha x) - 2\alpha^{100} f(x))(t) \geq \nu(x, 0)(t), \tag{7}$$

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for all $x \in E$ and which implies

$$\rho\left(\frac{f(\alpha x)}{\alpha^{100}} - f(x)\right)(t) = \rho\left(2f(\alpha x) - 2\alpha^{100}f(x)\right)(2^{\beta}\alpha^{100\beta}t) \geq \nu(x,0)(2^{\beta}\alpha^{100\beta}t), \tag{8}$$

for all $x \in E$. Substituting $x$ by $\alpha^{-1}x$ in (8), we obtain

$$\begin{aligned}\rho\left(\frac{f(\alpha^{-1}x)}{\alpha^{-100}} - f(x)\right)(t) &= \rho\left(\frac{f(x)}{\alpha^{100}} - f(\alpha^{-1}x)\right)\left(\frac{t}{\alpha^{100\beta}}\right) \\ &\geq \nu(\alpha^{-1}x,0)\left(2^{\beta}\alpha^{100\beta}N^{-1}\frac{Nt}{\alpha^{100\beta}}\right) \\ &\geq \nu(x,0)\left(2^{\beta}\alpha^{100\beta}N^{-1}t\right).\end{aligned} \tag{9}$$

From (8) and (9), we obtain

$$\rho\left(\frac{f(\alpha^{a}x)}{\alpha^{100a}} - f(x)\right)(t) \geq \Psi(x)(t) := \nu(x,0)\left(2^{\beta}\alpha^{100\beta}N^{\frac{a-1}{2}}t\right) \tag{10}$$

for all $x \in E$. Consider $P := h : E \to (X,\rho)|h(0) = 0$ and define $\eta$ on $P$ as follows,

$$\eta(h) = \inf l > 0 : \rho(h(x))(lt) \geq \Psi(x)(t),$$

for all $x \in E$. One can easily prove that $\eta$ modular on $N$ and indulges the $\Delta_2-$condition with $2^{\beta} = \kappa$ and Fatou property. Additionally, $N$ is $\eta-$complete, see[30]. Consider the mapping $R : P_{\eta} \to P_{\eta}$ as $RM(x) := \frac{M(\alpha^{a}x)}{\alpha^{100a}}$ for all $M \in P_{\eta}$.

Let $h, j \in P_{\eta}$ and $l > 0$ be an arbitrary constant with $\eta(h-j) \leq l$. From the definition of $\eta$, we get

$$\rho(h(x) - j(x))(lt) \geq \Psi(x)(t)$$

for all $x \in E$ and which implies

$$\begin{aligned}&\rho\left(Rh(x) - Rj(x)\right)(Nlt) \\ &\quad= \rho\left(\alpha^{-100a}h(\alpha^{a}x) - \alpha^{-100a}j(\alpha^{a}x)\right)(Nlt) \\ &\quad= \rho\left(h(\alpha^{a}x) - j(\alpha^{a}x)\right)\left(\alpha^{100\beta a}Nlt\right) \\ &\quad\geq \Psi(\alpha^{a}x)(\alpha^{100\beta a}Nt) \\ &\quad\geq \Psi(x)(t)\end{aligned}$$

for all $x \in E$. Hence $\eta(Rh - Rj) \leq N\eta(h-j)$, for all $h, j \in P_{\eta}$, which means, $R$ is a $\eta-$strict contraction. Replacing $x$ by $\alpha^{a}x$ in (10), we arrive

$$\rho\left(\frac{f(\alpha^{2a}x)}{\alpha^{100a}} - f(\alpha^{a}x)\right)(t) \geq \Psi(\alpha^{a}x)(t) \tag{11}$$

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for all $x \in E$ and therefore

$$\begin{aligned}
&\rho\left(\alpha^{-2(100a)} f(\alpha^{2a}x) - \alpha^{-100a} f(\alpha^a x)\right)(Nt) \\
&\quad = \rho\left(\alpha^{-100a} f(\alpha^{2a}x) - f(\alpha^a x)\right)(\alpha^{100\beta a} Nt) \\
&\quad \geq \Psi(\alpha^a x)(\alpha^{100\beta a} Nt) \geq \Psi(x)(t),
\end{aligned} \tag{12}$$

for all $x \in E$. Now

$$\begin{aligned}
&\rho\left(\frac{f(\alpha^{2a}x)}{\alpha^{2(100a)}} - f(x)\right)\left(2^\beta(Nt+t)\right) \\
&\geq \rho\left(\frac{f(\alpha^{2a}x)}{\alpha^{2(100a)}} - \frac{f(\alpha^a x)}{\alpha^{100a}}\right)(Nt) \wedge \rho\left(\frac{f(\alpha^a x)}{\alpha^{100a}} - f(x)\right)(t) \\
&\geq \Psi(x)(t)
\end{aligned} \tag{13}$$

for all $x \in E$. In (13), replacing $x$ by $\alpha^a x$ and $2^\beta(Nt+t)$ by $\alpha^{100\beta a} 2^\beta(N^2t + Nt)$, we arrive

$$\begin{aligned}
&\rho\left(\frac{f(\alpha^{3a}x)}{\alpha^{2(100a)}} - f(\alpha^a x)\right)\left(\alpha^{100\beta a} 2^\beta(N^2t + Nt)\right) \\
&\quad \geq \Psi(\alpha^a x)(\alpha^{100\beta j Nt}) \geq \Psi(x)(t)
\end{aligned} \tag{14}$$

for all $x \in E$. Therefore,

$$\rho\left(\frac{f(\alpha^{3a}x)}{\alpha^{3(100a)}} - \frac{f(\alpha^a x)}{\alpha^{100a}}\right)\left(2^\beta(N^2t + Nt)\right) \geq \Psi(x)(t), \tag{15}$$

for all $x \in E$ and which implies

$$\begin{aligned}
&\rho\left(\frac{f(\alpha^{3a}x)}{\alpha^{3(100a)}} - f(x)\right)\left(2^\beta(2^\beta(N^2t + Nt) + t)\right) \\
&\geq \rho\left(\frac{f(\alpha^{3a}x)}{\alpha^{3(100a)}} - \frac{f(\alpha^a x)}{\alpha^{100a}}\right)\left(2^\beta(N^2t + Nt)\right) \wedge \rho\left(\frac{f(\alpha^a x)}{\alpha^{100a}} - f(x)\right)(t) \\
&\geq \Psi(x)(t),
\end{aligned} \tag{16}$$

for all $x \in E$. Generalizing the above inequality, we arrive

$$\rho\left(\frac{f(\alpha^{am}x)}{\alpha^{100(am)}} - f(x)\right)\left((2^\beta N)^{m-1}t + 2^\beta \sum_{i=1}^{m-1}(2^\beta N)^{i-1}t\right) \geq \Psi(x)(t), \tag{17}$$

for all $x \in E$ and $m$ be a positive integer. Hence, we have

$$\begin{aligned}
\eta(R^m f - f) &\leq (2^\beta N)^{m-1} + 2^\beta \sum_{i=1}^{m-1}(2^\beta N)^{i-1} \\
&\leq 2^\beta \sum_{i=1}^{m}(2^\beta N)^{i-1} \leq \frac{2^\beta}{1 - 2^\beta N},
\end{aligned} \tag{18}$$

Now, one can easily prove that $\{R^m(f)\}$ is $\eta-$converges to $M \in P_\eta$, see[30]. Therefore, inequality (18) becomes

$$\eta(M-f) \leq \frac{2^\beta}{1-2^\beta N}, \tag{19}$$

which leads

$$\rho\left(M(x)-f(x)\right)\left(\frac{2^\beta}{1-2^\beta N}t\right) \geq \Psi(x)(t) = \nu(x,0)\left(2^\beta \alpha^{100\beta} N^{\frac{a-1}{2}} t\right), \tag{20}$$

for all $x \in E$ and hence, we have

$$\rho\left(M(x)-f(x)\right)\left(\frac{t}{\alpha^{100\beta} N^{\frac{a-1}{2}}(1-2^\beta N)}\right) \geq \nu(x,0)(t), \tag{21}$$

for all $x \in E$ and hence the inequality (6) holds. One can easily prove the uniqueness of $M$ by assuming $M'$ be another fixed point of $R$ implies that $M = M'$, see[30]. □

## 4. Conclusion

In this paper, author introduced a new hectic radical functional equation satisfied by the solution $f(x) = x^{100}$. Mainly, the authors obtained its general solution and investigated its generalized Hyers-Ulam-Rassias stability in $PM-$space by using fixed point theory.

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